Exercise 5E

1 a $y = 5x - 2$, $m = 5$
$15x - 3y + 9 = 0$
Parallel lines have the same gradient.
Rearrange the second equation to give:
$3y = 15x + 9$
$y = 5x + 3$, $m = 5$
The lines have the same gradients so they are parallel.

b $7x + 14y - 1 = 0$
$y = \frac{1}{2}x + 9$, $m = \frac{1}{2}$
Parallel lines have the same gradient.
Rearrange the first equation to give:
$14y = -7x + 1$
$y = -\frac{1}{2}x + \frac{1}{14}$, $m = -\frac{1}{2}$
The lines have different gradients so they are not parallel.

c $4x - 3y - 8 = 0$
$3x - 4y - 8 = 0$
Parallel lines have the same gradient.
Rearrange the first equation to give:
$3y = 4x - 8$
$y = \frac{4}{3}x - \frac{8}{3}$, $m = \frac{4}{3}$
Rearrange the second equation to give:
$4y = 3x - 8$
$y = \frac{3}{4}x - 2$, $m = \frac{3}{4}$
The lines have different gradients so they are not parallel.

2 The gradient of r is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{6-1} = \frac{4}{5}$$

The gradient of s is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9-(-3)}{20-5} = \frac{12}{15} = \frac{4}{5}$$

2 The gradients are equal, so the lines are parallel.

3 If $A(-6, 2)$, $B(4, 8)$, $C(6, 1)$ and $D(-9, -8)$ are coordinates of a trapezium, AB is parallel to CD or BC is parallel to DA.
Parallel lines have the same gradient.

$$\text{The gradient of } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{4-(-6)} = \frac{6}{10} = \frac{3}{5}$$

$$\text{The gradient of } CD = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-1}{-9-6} = \frac{-9}{-15} = \frac{3}{5}$$

Gradient of AB = gradient of CD.
AB is parallel to CD, therefore $ABCD$ is a trapezium.

4 The line is parallel to $y = 5x + 8$, so $m = 5$.
The line intercepts the y-axis at $(0, 3)$, so $c = 3$.
Using $y = mx + c$, the equation of the line is $y = 5x + 3$.

5 The line is parallel to $y = -\frac{2}{5}x + 1$, so $m = -\frac{2}{5}$.
The line intercepts the y-axis at $(0, -4)$, so $c = -4$.
Using $y = mx + c$, the equation of the line is $y = -\frac{2}{5}x - 4$.
Multiply each term by 5:

$$5y = -2x - 20$$
$$2x + 5y = -20$$
$$2x + 5y + 20 = 0$$

6
$$3x + 6y + 11 = 0$$
$$6y + 11 = -3x$$
$$6y = -3x - 11$$
$$y = -\frac{3}{6}x - \frac{11}{6}$$
$$y = -\frac{1}{2}x - \frac{11}{6}$$

The line is parallel to $y = -\frac{1}{2}x - \frac{11}{6}$, so $m = -\frac{1}{2}$.
The line intercepts the y-axis at $(0, 7)$, so $c = 7$.
Using $y = mx + c$, the equation of the line is $y = -\frac{1}{2}x + 7$.

7
$$2x - 3y - 1 = 0$$
$$2x - 1 = 3y$$
$$3y = 2x - 1$$
$$y = \frac{2}{3}x - \frac{1}{3}$$

The line is parallel to $y = \frac{2}{3}x - \frac{1}{3}$, so $m = \frac{2}{3}$.
The line intercepts the y-axis at $(0, 0)$, so $c = 0$.
Using $y = mx + c$:

$$y = \frac{2}{3}x + 0$$
$$y = \frac{2}{3}x$$

8 The gradient of a line parallel to $y = 4x + 1$ is 4.

$$y - y_1 = m(x - x_1)$$
$$y - 7 = 4(x - (-2))$$
$$y - 7 = 4(x + 2)$$
$$y - 7 = 4x + 8$$
$$y = 4x + 15$$
$$0 = 4x + 15 - y$$
$$4x - y + 15 = 0$$

The equation of the line is $4x - y + 15 = 0$.